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TIME SERIES ANALYSIS AND MULTIVARIATE STATISTICAL ANALYSIS

FINAL REPORT

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Principal Investigator

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Research was carried out mainly in the areas of time series analysis and multivariate statistical analysis. The most important results in the first area apply to autoregressive and moving average processes. In the second area emphasis was on parameter consistency and elliptically contoured distributions.

The autoregressive model has been developed in a simple, rigorous, and coherent way, with explicit expressions for the coefficients of the moving average representation and the autocovariance sequence. The conditions on the roots of the associated polynomial equation and properties of the disturbances are implied by stationarity and an independence assumption. A general martingale limit theorem was obtained and applied to autoregressive processes, giving asymptotic results stronger than previous results. The Kalman filter has been used to obtain a sequence of estimates of the parameters of an autoregressive model available as the data are collected; the technique is applicable to panel studies as well as to single time series. Bayes estimates for a prior exchangeable distribution of parameters and for a prior autoregressive distribution of parameters were found. The multivariate autoregressive model has been used to study causality.

To estimate the parameters of the moving average model 16 different iterative procedures have been devised. These involve alternative parametrizations, time and frequency domain representations, Newton-Raphson and scoring approaches, and use of likelihoods and concentrated likelihoods. Properties of the likelihood function, as well as the estimates, have been derived.

"Parameter consistency" of a test means that the power of the test tends towards 1 as the true value of the parameter moves away from the hypothesized value. A complete analysis of this property has been carried out in the multivariate analysis of variance and applied to the conventional tests. They are all parameter consistent except for the Bartlett-Nanda-Pillai trace test: in that case consistency depends on the significance level, dimensionality, and numbers of degrees of freedom. The relationship to the usual notation of consistency has been carried out.

It has been shown that in a class of elliptically contoured distributions  $F$ -tests in the analysis of variance are valid even though the quadratic forms in the numerator and denominator of the  $F$ -ratio are not independent; see discussion of "What is an analysis of variance?" by T. P. Speed. Conditions for Cochran's theorem have been relaxed. The normal distribution within the class of elliptically contoured distributions has been characterized by the distribution of a quadratic form.



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## Scientific Personnel

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Bruce Newell  
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